Perturbation growth and cosmic microwave background anisotropies in the string-like matter dominated universe

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Abstract. The growth of small perturbations and the temperature correlation function of cosmic background radiation induced by the perturbations in the isotropic and homogeneous universe with arbitrary spatial curvature, filled with nonrelativistic and so-called string-like matter is calculated. It is shown that both magnitudes are insensitive to the eventual existence of the string-like matter in the universe. The widespread opinion that the string-like matter dominated universe suppresses the growth of perturbation is verified.

1. Introduction

The discrepancy between observationally determined value of the energy density parameter $\Omega_{\mathrm{obs}} = 0.1 \div 0.3$ and the theoretically favoured one $\Omega \approx 1$ (inflationary scenario) suggests that the Universe can be filled with some kind of matter which is distributed more smoothly than the baryonic matter. The smooth component of the Universe must be nonbaryonic but it can be of nonrelatyvistic (e.g. massive neutrinos), relativistic, or more exotic (e.g. Λ-term) type. The problem appears in the case of nonrelativistic and relativistic matter dominated universe since in both cases the age of the universe becomes too short (shorter than the age of the oldest stars and globular clusters). This is the main reason why cosmological models with the Λ-term are in recent years so seriously taken into account. However, the constant Λ-term is not the only possibility which resolves the so called "Ω-problem" giving, at the same time, sufficiently large age of the universe. Some people consider the Λ-term varying in time according to the law $\Lambda \propto t^{-2}$ [1,2] or $\Lambda \propto R^{-2}$ [3,4] (in some cases both behaviours coincide). Such time-dependent Λ-term allows to explain its present extremely small value comparing with the natural value close to the Planck epoch. We note that the global texture in a closed universe [5] and a network of cosmic strings [6,7] obeys the same law of variation ($\rho \propto R^{-2}$). We call the exotic form of matter scaling according to this law – string-like matter. In the present paper we discuss perturbation growth and anisotropies of the cosmic microwave background (CMB) in the string-like matter dominated universe.

The influence of, what we call, string-like matter on the evolution of the universe and on astrophysical formulae was examined by several authors [8-13]. Perturbation growth in such a model was also discussed [14]. It is well known that the perturbations cannot grow in the curvature dominated universe. Since the energy density of the string-like matter scales with the expansion in the same way as the curvature term one asserts that the perturbations cannot grow in the string-like matter dominated universe either. We find this generally accepted statement not quite correct and one of the purposes of the paper is to clear up this point. The second purpose of the paper is to check whether the eventual existence of the string-like matter in the universe may have an impact on the large-angular-scale anisotropies of the CMB. In the previous paper [12] we showed that the string-like matter enlarges the angle at which we observe the anisotropies of the CMB. One can say that the string-like matter acts as a sort of magnifying glass apparently increasing the angular size of anisotropy. The question we address in the present paper is whether the string-like matter influes the amplitude of the anisotropy.

In the next section we calculate the growth of small perturbations in the homogeneous

and isotropic universe with arbitrary spatial curvature filled with nonrelativistic and string-like matter. Section 3 is devoted to evaluation of the temperature correlation function of cosmic microwave background radiation induced by the perturbations in the model under consideration. We realize that neither the growth of perturbations nor the temperature correlation function of the CMB is sensitive to the eventual existence of the string-like matter in the universe.

2. Perturbation growth in the universe filled with nonrelativistic and string--like matter

If we assume that the components of the universe do not interact with each other the Friedman equation can be written in the form

$$\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 = \frac{C_m}{R^3} + \frac{C_s}{R^2} - \frac{k}{R^2}, \tag{2.1}$$

where $k = 0, \pm 1$ is a curvature index and C_m and C_s are some constants which can be expressed in terms of astronomical parameters [10]

$$C_m = H_0{}^2 \Omega_{m0}, \tag{2.2a}$$

$$C_s = H_0{}^2 \Omega_{s0}, \tag{2.2b}$$

$$k = H_0{}^2 (\Omega_{m0} + \Omega_{s0} - 1). \tag{2.2c}$$

H_0 is a present value of the Hubble constant and Ω_{m0} and Ω_{s0} are energy density parameters of nonrelativistic and string-like matter respectively. We use scale factor normalization such that at the present epoch $R(t_0) = 1$. With the help of expressions (2.2) the Friedman equation reads

$$\left(\frac{dR}{dt}\right)^2 = \frac{\Omega_{m0} H_0{}^2}{R} - H_0{}^2 (\Omega_{m0} - 1). \tag{2.3}$$

Note that the string-like matter energy density parameter Ω_{s0} does not explicitly appear in the above equation. It means that the string-like matter does not affect the dynamics of the universe. It follows from the relation (2.2c) that Ω_{s0} does affect only the curvature. This well known property will be crucial for our further considerations.

Similar form of the curvature term and the string-like matter term in the Friedman equation suggests introducing, what we call, "effective curvature term" k'/R^2, where

$$k' \equiv k - C_s = H_0{}^2 (1 - \Omega_{m0}). \tag{2.4}$$

This term affects directly the dynamics of the universe. The curvature term and the string-like matter term separately do not.

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Let us assume that the string-like matter is distributed smoothly in the universe (at a large scale) i.e. is not affected by a local perturbation of nonrelativistic matter $\delta\rho(t,\vec{x}) = \rho(t,\vec{x}) - \bar{\rho}(t)$. Then the time evolution of the density contrast $\delta \equiv \delta\rho/\bar{\rho}$ in the linear regime ($\delta \ll 1$) is described by the equation

$$\ddot{\delta} + 2\frac{\dot{R}}{R}\dot{\delta} - \frac{3}{2}\frac{\Omega_{m0}H_0{}^2}{R^3}\delta = 0. \tag{2.5}$$

Since the string-like matter forms a smoth background it does not explicitly enter the above equation [15]. Its solution (growing mode) written down as a function of redshift, up to some multiplicative constant reads [16]

$$\delta(z) = \frac{1 + 2\Omega_{m0} + 3\Omega_{m0}z}{(1 - \Omega_{m0})^2}$$
$$- \frac{3}{2}\frac{\Omega_{m0}(1+z)}{(1 - \Omega_{m0})^{5/2}}\sqrt{1 + \Omega_{m0}z}\ln\frac{\sqrt{1 + \Omega_{m0}z} + \sqrt{1 - \Omega_{m0}}}{\sqrt{1 + \Omega_{m0}z} - \sqrt{1 - \Omega_{m0}}}. \tag{2.6}$$

Of course this is exactly the same solution as in the case without the string-like matter as it could have been expected looking at the form of the equation (2.5). From the solution it follows that for fixed z the growth of perturbation in the linear regime is totally determined by the nonrelativistic matter energy density parameter Ω_{m0}. The string-like matter energy density parameter Ω_{s0} has nothing to do with the growth $\delta(z)$. For fixed Ω_{m0} the growth is the same independently of whether $\Omega_{s0} = 0$ or $\Omega_{s0} \gg \Omega_{m0}$. The same conclusion can be deduced from the form of the equation (2.1) or (2.3). In the case without the string-like matter perturbations cease to grow if only the curvature term starts to dominate, i.e. for $|k|/R^2 > C_m/R^3$. But in the case under consideration the curvature term is replaced by the "effective curvature term" k'/R^2, which is a difference of the curvature term and the string-like matter term. In this difference Ω_{s0} cancels. As a result the redshift at which the perturbation ceases to grow is independent of the value of Ω_{s0}

$$z_c = \frac{1 - \Omega_{m0}}{\Omega_{m0}} - 1. \tag{2.7}$$

3. Temperature correlation function in the model with nonrelativistic and string-like matter

The perturbation growth calculated in the previous section by local change of gravitational potential induces anisotropy in the cosmic microwave background radiation. This anisotropy can be calculated in several ways. In the present paper we are using a method

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proposed by Górski *et al* [17]. In this method an expression for the temperature correlation function $C(\theta)$ involves a double integral along the lines of sight, separated by the angle θ, to the last scattering surface over the second derivative of the growth rate factor for adiabatic density fluctuation in nonrelativistic matter combined with simple integral transforms of the power spectrum of inhomogeneity

$$C(\theta) = \int \int_0^{\eta_0 - \eta_e} ds_1 ds_2 \ddot{\delta}(\eta_e + s_1)\ddot{\delta}(\eta_e + s_2) \left[\psi_\parallel(y) \cos\theta + \psi_-(y)\frac{y_1 y_2}{y^2} \sin^2\theta \right]. \qquad (3.1)$$

$\ddot{\delta}(\eta)$ is the second derivative of the perturbation δ with respect to the conformal time η defined $d\eta = H_0 dt/R(t)$. Using the evolution equation (2.5) written down in a form with η-variable

$$\ddot{\delta}(\eta) + \frac{\dot{R}(\eta)}{R(\eta)}\dot{\delta}(\eta) - \frac{3}{2}\Omega_{m0}\frac{\delta(\eta)}{R(\eta)} = 0 \qquad (3.2)$$

$\ddot{\delta}(\eta)$ can be expressed as

$$\ddot{\delta}(\eta) = (1 - \Omega_{m0})\left(\frac{9}{\cosh\eta - 1} + 2\right)\delta(\eta) - 1, \qquad (3.3)$$

where the density contrast δ as a function of η is

$$\delta(\eta) = \frac{1}{H_0{}^2(1 - \Omega_{m0})}\left[1 + \frac{6}{\cosh\eta - 1} + \frac{3}{\cosh\eta - 1}\left(\frac{2}{\cosh\eta - 1} + 1\right)^{1/2}\right.$$
$$\left. \times \ln\frac{\left(\frac{2}{\cosh\eta - 1} + 1\right)^{1/2} - 1}{\left(\frac{2}{\cosh\eta - 1} + 1\right)^{1/2} + 1}\right]. \qquad (3.4)$$

In order to find $\delta(\eta)$ in the above form we integrated the Friedman equation

$$\left(\frac{dR}{d\eta}\right) = \Omega_{m0}R + (1 - \Omega_{m0})R^2 \qquad (3.5)$$

and the solution is

$$R(\eta) = \frac{\Omega_{m0}}{2(1 - \Omega_{m0})}(\cosh\eta - 1). \qquad (3.6)$$

η_0 in the upper limit of the integration in (3.1) is the value of the time parameter corresponding to the present moment and can be calculated from (3.6) by putting $R(\eta_0) = 1$.

$$\eta_0 = \text{arcosh}\frac{2 - \Omega_{m0}}{\Omega_{m0}}. \qquad (3.7a)$$

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η_e corresponds to the last scattering surface and can be found similarly by putting $R(\eta_e) = 1/(z+1)$, where z is the redshift of the surface ($z \approx 1100$).

$$\eta_e = \operatorname{arcosh}\left[\frac{2(1-\Omega_{m0})}{\Omega_{m0}(z+1)} + 1\right]. \tag{3.7b}$$

y_1, y_2 and y are defined

$$y_1 = c(\eta_0 - \eta_e - s_1), \tag{3.8a}$$

$$y_2 = c(\eta_0 - \eta_e - s_2), \tag{3.8b}$$

$$y^2 = {y_1}^2 + {y_2}^2 - 2y_1 y_2 \cos\theta. \tag{3.8c}$$

The functions $\psi_\|(y)$ and $\psi_-(y)$ are given by

$$\psi_\|(y) = \frac{1}{2\pi^2 c^2} \int_0^\infty dk P(k) \frac{dj_1(ky)}{dky}, \tag{3.9a}$$

$$\psi_-(y) = \frac{1}{2\pi^2 c^2} \int_0^\infty dk P(k) j_2(ky), \tag{3.9b}$$

where j_1 and j_2 are spherical Bessel functions

$$j_1(x) = \frac{\sin x - x \cos x}{x^2}, \tag{3.10a}$$

$$j_2(x) = \frac{(3-x^2)\sin x - 3x\cos x}{x^3}. \tag{3.10b}$$

Finally $P(k)$ is the perturbation power spectrum which is choosen in a standard form [18,19]

$$P(k) = \frac{Ak^n}{(1 + \alpha k + \beta k^{3/2} + \gamma k^2)^2}, \tag{3.11}$$

where

$$\alpha = 170(\Omega_{m0}h)^{-1}\text{km/s}, \tag{3.12a}$$

$$\beta = 9 \times 10^3 (\Omega_{mo}h)^{-3/2}(\text{km/s})^{3/2}, \tag{3.12b}$$

$$\gamma = 10^4 (\Omega_{m0}h)^{-2}(\text{km/s})^2. \tag{3.12c}$$

h is the Hubble constant in units of $100 \text{ kms}^{-1}\text{Mpc}^{-1}$. The discrepancy between the numerical values of the constanys α, β, γ given in original papers and the values given above follows from the different choice of units for k. Usually one chooses $[k] = \text{Mpc}^{-1}$. Following Górski *et al.* we choose $[k] = (\text{km/s})^{-1}$. The latter units are obtained from the

former ones by dividing them by the factor $H_0 = 100 \ h\text{km/sMpc}$. n is the spectral index usually taken as $n \approx 1$ (Harrison-Zel'dovich spectrum). The constant A can be found from the normalization condition [17]

$$1 = \frac{\delta^2(\eta_0)}{2\pi^2} \int_0^\infty dk k^2 P(k) \left[\frac{3j_1(800k)}{800k} \right]^2 . \tag{3.13}$$

$\delta(\eta_0)$ can be obtained from (2.6) by putting $z = 0$.

In order to find the temperature correlation function $C(\theta)$ in the model, the expressions for $\eta_0, \eta_e, \delta(\eta)$ and $\ddot{\delta}(\eta)$ should be now substituted into the formula (3.1). Numerical determination of the function $C(\theta)$ is long-lasting because the multiple integration has to be carried out. But even without integration some conclusions may be deduced. It seems that the crucial one is that the temperature correlation function $C(\theta)$, or in the consequence, the anisotropy amplitude $\delta T/T(\theta) = [2(C(0) - C(\theta))]^{1/2}$ does not depend on the amount of string-like matter in the universe. The amplitude is the same in the model with and without the string-like matter.

4. Conclusions

In a widespread opinion the suppression of the perturbation growth is a most important flaw of the string-like matter dominated universe practically eliminating it as a model of the real universe.

One can easily show that in the model with two components (nonrelativistic and string-like matter) noninteracting with each other the string-like matter starts to dominate at the redshift

$$z_s = \frac{\Omega_{s0}}{\Omega_{m0}} - 1. \tag{4.1}$$

Note that unless $k = 0$ the value of z_s has nothing to do with the value of the redshift z_c at which the "effective curvature term" starts to dominate

$$z_c = \frac{1 - \Omega_{m0}}{\Omega_{m0}} - 1. \tag{4.2}$$

Formally it might happen that the universe is dominated by the string-like matter and the perturbations still grow because $z_s > z_c$ (in the case when $\Omega_{s0} + \Omega_{m0} - 1 > 0$).

In the paper we showed explicitly that the perturbation growth in the linear regime is totally independent of the amount of the string-like matter in the universe. The growth is completely determined by the amount of nonrelativistic matter. For example if we fix $\Omega_{m0} < 1$, then for $\Omega_{s0} < 1 - \Omega_{m0}$ $(k = -1)$ or $\Omega_{s0} = 1 - \Omega_{m0}$ $(k = 0)$ or $\Omega_{s0} > 1 - \Omega_{m0}$ $(k = 1)$ the total perturbation growth is the same. One can say that the perturbation growth

does not feel the existence of the string-like matter. What distinguishes all three cases is the spatial curvature. At the first sight it seems strange that the growth rate of the perturbation does not depend on the curvature of the universe. This, however, should not be surprising because the curvature term and the string-like matter term enter the dynamical equations always together as an "effective curvature term" and the curvature in the extracted form does not appear.

Since the calculation of the temperature correlation function of the CMB requires practically the same equations as the calculation of the perturbation growth the amplitude of the anisotropy of the CMB does depend neither on the curvature nor on the Ω_{s0}-parameter either. It means that the measurement of the amplitude of the anisotropy $\delta T/T(\theta)$ does not say anything about the eventual existence of the string-like matter in the universe.

Conluding we would like to stress once more that the formation of structure in the universe with or without the string-like matter proceeds in the same way. There is no suppression of the perturbation growth in the string-like matter dominated universe. Moreover such exotic matter has no effect on the value of the anisotropy amplitude $\delta T/T(\theta)$.

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